RELATIVISTIC HAMILTONIAN

classically Hamiltonian function H is given by

$H = \sum P_r \dot{q}_r - L$ where H is the total energy of the system H, in relativity can be defined as

From (7) classically

$$H = \sum \frac{\partial T^*}{\partial \dot{q}_r} \dot{q}_r - L \quad \left\{ P_r = \frac{\partial T}{\partial \dot{q}_r} \text{ and } T^* \right.$$

relativistic value

Putting relativistic Lagrangian value from (5) and (6) we have

$$H = \sum \frac{\partial T^*}{\partial \dot{q}_r} \dot{q}_r - m_0 c^2 \left(1 - \sqrt{1-\alpha^2}\right) + V$$

Putting values of $\frac{\partial T^*}{\partial \dot{x}}$ etc, from eqn (4) we have

$$[q_r \equiv q_r (x, y, z)]$$

$$H = \frac{m_0}{\sqrt{(1-\alpha^2)}} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - m_0 c^2 (1 - \sqrt{(1-\alpha^2)}) + V$$

$$= m_0 c^2 \left[\frac{\alpha^2}{\sqrt{(1-\alpha^2)}} - 1 + \sqrt{(1-\alpha^2)} \right] + V$$

$$= m_0 c^2 \left[\frac{1 - \sqrt{(1-\alpha^2)}}{\sqrt{(1-\alpha^2)}} \right] + V$$

$$= m_0 c^2 \left[\frac{1}{\sqrt{(1-\alpha^2)}} - 1 \right] + V \quad \text{——— (1)}$$

$H = T + V$ from (1)

Hence relativistic Hamiltonian like classical Hamiltonian also gives the total energy of the system

Also

$$P_x^2 + P_y^2 + P_z^2 = \frac{m_0^2}{1-\alpha^2} (\dot{x}^2 + \dot{y}^2 + \dot{z}^2) \text{ by (4)}$$

$$= \frac{m_0^2 u^2}{1-\alpha^2} = \frac{m_0^2 c^2 \alpha^2}{1-\alpha^2}$$

$$\therefore P_x^2 + P_y^2 + P_z^2 = \frac{m_0^2 c^2 \alpha^2}{1-\alpha^2} = m_0^2 c^2 \left[\frac{1}{1-\alpha^2} - 1 \right]$$

$$\frac{1}{1-\alpha^2} = \frac{m_0^2 c^2 + P_x^2 + P_y^2 + P_z^2}{m_0^2 c^2}$$

(1) gives

$$H = m_0 c^2 \left[\sqrt{\left(\frac{m_0^2 c^2 + P_x^2 + P_y^2 + P_z^2}{m_0^2 c^2} \right)} - 1 \right]$$

$$H = c \sqrt{(m_0^2 c^2 + P_x^2 + P_y^2 + P_z^2)} - m_0 c^2$$

It is required relativistic Hamiltonian

Hamiltonian canonical equation are

$$\dot{x} = \frac{\partial H}{\partial P_x} \quad \text{and} \quad \dot{P}_x = \frac{\partial H}{\partial x}$$

using (3)

$$\dot{x} = \frac{cP_x}{\sqrt{(m_0^2c^2 + P_x^2 + P_y^2 + P_z^2)}}$$

$$= \frac{cP_x\sqrt{(1-\alpha^2)}}{m_0c} \quad \text{by (2)}$$

$$\therefore P_x = \frac{m_0\dot{x}}{\sqrt{(1-\alpha^2)}}$$

It is the correct relativistic momentum

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